Pergamon

Int. J. Heat Mass Transfer. Vol. 41, No. 10, pp. 1243–1253, 1998

Printed in Great Britain
0017–9310/98 $19.00+0.00

PII: S0017–9310(97)00209–3

Heat transfer optimization in internally finned tubes under laminar flow conditions

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(*Received* 21 *March* 1997 *and in final form* 23 *June* 1997)

Abstract—In the present work the problem of optimizing the geometry of internally finned tubes in order to enhance the heat transfer under laminar flow conditions is studied. The velocity and temperature distributions on the finned tube cross-section are determined with the help of a finite element model, and a global heat transfer coefficient is calculated. A polynomial lateral profile is proposed for the fins and the geometry is optimized in order to make the heat transferred per unit of tube length or surface as high as possible for a given weight and for a given hydraulic resistance. Finally, the optimum fin profiles obtained by means of a genetic algorithm are shown for different situations. © 1998 Elsevier Science Ltd.

INTRODUCTION

In order to enhance the heat transfer, finned surfaces are commonly used in many engineering sectors. To remove high heat fluxes from very small components, the need to reduce the weight or the volume of thermal dissipator systems has become even more important, particularly in new applications, such as in the electronic industry [1, 2] or in the compact heat exchanger field [3]. Therefore, many researchers have studied the problem of optimizing the shape of the finned surfaces in order to increase the heat transfer effectiveness and reduce the weight of dissipators, but for many situations a definitive solution has not yet been found.

An optimum profile for longitudinal fins was proposed by Schmidt [4], who suggested the adoption of a parabolic shape. Such a suggestion was confirmed by Duffin [5] on the basis of a rigorous variational method. Afterwards, many authors contested Schmidt's conclusion [6, 7], which was correct from the point of view of the utilized model, but scarcely corresponding to the real phenomenon characteristics. Many fin profiles have later been proposed, mainly parabolic or triangular, leaving, however, uncertainty regarding the structural integrity of the fins [8]. Undulate and parabolical-undulate fin profiles have recently been proposed and demonstrated as having a noticeably improved effectiveness [9–11].

In a previous work [12] we presented a genetic algorithm which was able to optimize the heat transfer through longitudinal fins, whose lateral profile was described by a polynomial function, by considering a constant convective heat transfer coefficient. Such an assumption has been followed by many authors who studied the problem of optimizing the lateral profile of longitudinal fins, but in some situations the variations in the local convective heat transfer coefficient induced by the alterations of the fin shape cannot be correctly neglected. In particular, in internally finned tubes, in which a coolant fluid passes through in laminar flow, the alteration of the lateral fin profile can cause great changes in the local convective heat transfer coefficient.

In the present work we then study the problem of optimizing the geometry of internally finned tubes in which a heat flux uniformly imposed on the external surface is dissipated under conditions of laminar coolant flow. Such a problem finds practical application in the compact heat exchanger field or in the electronic component cooling sector, when the coolant velocity must be reduced in order to lower the noisiness of the devices, to avoid excessive power dissipations or to prevent the miniaturized structures from large pressure gradients. To study this problem, a mathematical model which is able to take the thermofluidodynamical alterations induced by changes in the fin profile into account is proposed. Moreover, some criteria for optimizing the performances of finned tubes, paying particular attention to the weight and the hydraulic resistance, are discussed. Finally, an appropriate genetic algorithm is utilized in order to find the optimum geometry in different situations.

THE FINNED TUBE MODEL

Let us consider a tube with internal fins, which are identical and have an axial symmetrical cross-section

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NOMENCLATURE

a	height of the fins [m]
c_p	specific heat capacity of the coolant [J kg^{-1} K^{-1}]
E_c	compared effectiveness
f	fin profile angle as a function of r [rad]
F'_v	viscous force per unit of length [N m^{-1}]
g_{Aik}	elements of the surface averaging matrix [m^2]
g_{Mik}	elements of the momentum transportation matrix
g_{Hik}	elements of the heat transportation matrix
h	global heat transfer coefficient [W m^{-2} K^{-1}]
k_c	thermal conductivity of the coolant [W m^{-1} K^{-1}]
l_i	perimeter crossed by q'' around the ith knot [m]
M	scale factor depending on the hydraulic resistance
n	polynomial order
Nu_e	equivalent Nusselt number
p	generalized pressure [N m^{-2}]
q''	heat flux per unit of surface [W m^{-2}]
q'_{k_i}	conductive heat flux which enters the ith knot per unit of length [W m^{-1}]
q'_{c_i}	convective heat flux which leaves the ith knot per unit of length [W m^{-1}]
r	radial coordinate [m]
r_i	radial coordinate of the ith knot [m]
R	internal radius [m]
R'_{p_i}	resultant of the pressure forces acting on the ith knot per unit of length [N m^{-1}]
R'_{v_i}	resultant of the viscous forces acting on the ith knot per unit of length [N m^{-1}]
s	unfinned wall thickness [m]
t_i	temperature of the ith knot [K]
T_b	bulk temperature of the coolant [K]
T_c	temperature of the coolant [K]
T_f	temperature of the finned tube [K]
T_{max}	maximum temperature on the external surface [K]
$\mathbf{u}$	coolant velocity [m s^{-1}]
u_i	coolant velocity of the ith knot [m s^{-1}]
w_i	coolant volume flow rate of the ith knot [m^3 s^{-1}]
w_t	total coolant volume flow rate [m^3 s^{-1}]
z	longitudinal coordinate [m].

Greek symbols

α	normalized height of the fins
β	angle between two symmetry axes [rad]
γ	ratio between finned tube and coolant thermal conductivity
η	normalized radial coordinate
θ	angular coordinate [rad]
θ_i	angular coordinate of the ith knot [rad]
μ	dynamic viscosity [kg m^{-1} s^{-1}]
ξ	normalized area of the fin cross-section
ρ	coolant density [kg m^{-3}]
σ	normalized unfinned wall thickness
$\bar{\sigma}$	normalized average wall thickness
ϕ	fin profile angle as a function of η [rad]
ϕ_i	fin profile describing parameters
ψ_i	polynomial coefficients.

[Fig. 1(a)]. A heat flux q'' is uniformly imposed on the external surface. Moreover, a coolant passes through the tube in laminar flow.

The heat transfer performances of the system can be determined by studying a portion of it delimited by two symmetry axes [Fig. 1(b)]. Let us choose a cylindrical coordinate system with the z-axis directed as the coolant flow. Let a be the fin height in the radial direction and $f(r)$ an arbitrary function of the radial coordinate r which provides the value of the angular coordinate θ on the lateral fin profile. Moreover, let R be the internal radius, s the unfinned tube wall thickness and β the angle between the symmetry axes.

Supposing that the system is in steady state and the natural convection is negligible with regard to the forced one, then where the velocity profile is completely developed the velocity vector $\mathbf{u}$ is parallel to the z-axis and is constant in the z-direction. Assuming uniform fluid properties and negligible viscous dissipation within the fluid, the coolant flow is then described by the following equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=\frac{1}{\mu}\frac{\mathrm{d}p}{\mathrm{d}z} \quad (1)$$

μ being the dynamic viscosity and p the generalized pressure, which includes the gravitation potential. Equation (1) must be integrated by imposing as boundary conditions that on the contact surfaces with the solid the velocity is null and on the symmetry axes and center the partial derivative of the velocity in the normal direction is null.

Where the thermal profile is fully developed, since the heat flux q'' is uniform, the temperatures of the fluid and of the solid linearly change with the z coordinate. The conductive heat flux in the z-direction is constant and can be neglected in thermal power balances. Therefore, the temperature distribution in the coolant is described by the following equation:

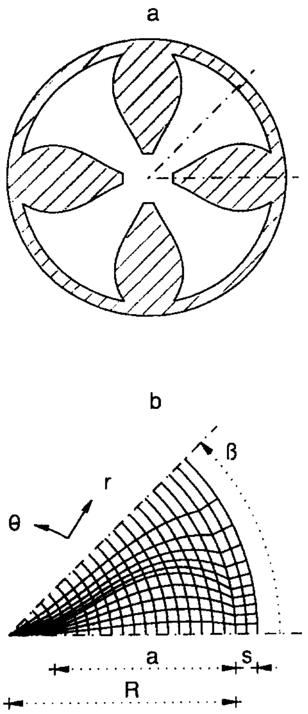

Fig. 1. Finned tube geometry: cross-section (a), subdivision of a portion of the cross-section in finite elements (b).

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_c}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T_c}{\partial \theta^2}=\frac{\rho c_p}{k_c}\mathbf{u}\frac{\partial T_c}{\partial z} \tag{2}$$

ρ being the density, c_p the specific heat and k_c the thermal conductivity of the coolant. The temperature distribution in the finned tube is instead described by Laplace's equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_f}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T_f}{\partial \theta^2}=0. \tag{3}$$

Equations (2) and (3) must be integrated by imposing as boundary conditions that on the contact surface between the solid and the fluid T_c is identical to T_f and the heat flux in the normal direction in the solid is identical to that in the fluid, on the symmetry axes and center the heat fluxes in the normal direction are null and on the external tube surface the heat flux in radial direction is equal to $-q''$. Moreover, the value of the temperature in one point of the section is required. Since the problem is very complex, it is convenient to determine the velocity and temperature distributions numerically, using for example a finite element method.

By opportunely locating some knots, we can subdivide the portion of the cross-section of the finned tube into an array of elements delimited by two concentric arches and two segments. In the center of the tube an element with the form of a circle sector can be located: suppose that in this element changes in the coolant velocity and temperature are negligible. An approximate example of subdivision is shown in Fig. 1(b). A large grid has been drawn for better comprehension. The elements of the coolant which are near the lateral fin surface can be more closely spaced in the θ-direction, since in the boundary region higher changes are expected in the velocity and in the temperature. In the r-direction the elements should always be closely spaced, in order to follow the fin profile without excessive distortions.

Let the velocity vector in each element of the coolant be approximated by an interpolation of the values u_i, which it assumes in the four knots of the element:

$$\mathbf{u}(r,\theta)=\sum_i \frac{\ln r-\ln r_{j(i)}}{\ln r_i-\ln r_{j(i)}}\frac{\theta-\theta_{k(i)}}{\theta_i-\theta_{k(i)}}u_i \tag{4}$$

r_i, r_j, θ_i and θ_k being knot coordinates. The element can be subdivided in four subelements by joining the middle points of the opposite sides. On the subdividing lines the viscous force which interacts between the parts of the element for a unitary length of the tube can be calculated as follows:

$$F_v'=\mu\sum_i \frac{\int[\theta-\theta_{k(i)}]\,\mathrm{d}z-\int[\ln r-\ln r_{j(i)}]\,\mathrm{d}(\ln r)}{[\ln r_i-\ln r_{j(i)}][\theta_i-\theta_{k(i)}]}u_i. \tag{5}$$

The resultant of all the viscous forces which act on the subelements which are around a knot can be written as

$$R'_{v_i}=\mu\sum_k g_{Mik}u_k \tag{6}$$

where the k index is extended to the knot i and to those which are around it. The parameters g_{Mik} depend on the coordinate of these knots and of those which delimit the subelements around the knot i. For the knots on the symmetry axes and around the tube centre g_{Mik} must be calculated by taking the conditions of null momentum flux into account. Since the flow is steady, the viscous resultant must be balanced by the resultant of the pressure forces:

$$R'_{p_i}=S_i\frac{\mathrm{d}p}{\mathrm{d}z} \tag{7}$$

where S_i is the total transversal surface of all the subelements around the knot i. Letting R'_{v_i} be equal to R'_{p_i} for all the knots, one obtains the following system:

$$\mathbf{G}_M*\mathbf{U}=\frac{1}{\mu}\frac{\mathrm{d}p}{\mathrm{d}z}\mathbf{A} \tag{8}$$

having gathered in matrix $\mathbf{G}_M$ the elements g_{Mik}, in vector $\mathbf{U}$ the velocities u_i and in vector $\mathbf{A}$ the surfaces S_i. Vector $\mathbf{U}$ can now be partitioned by grouping all the known terms in the subvector $\mathbf{U}_1$ and the unknowns in $\mathbf{U}_2$. Matrix $\mathbf{G}_M$ and vector $\mathbf{A}$ must be consequently partitioned. From eqn (8) we then obtain

$$\mathbf{G}_{M22}*\mathbf{U}_2=\frac{1}{\mu}\frac{\mathrm{d}p}{\mathrm{d}z}\mathbf{A}_2-\mathbf{G}_{M21}*\mathbf{U}_1. \tag{9}$$

By solving eqn (9) the velocity distribution in the portion of the conduit section is determined.

Let the temperature in each element be approximated by an interpolation of the temperatures t_k of the knots, with the same form factors of eqn (4). Due to the analogy between the momentum transfer and the heat transfer, the conductive heat flux which enters the subelements around a knot for a unitary tube length is then

$$q'_{k_i} = k_c \sum_k g_{Hik} t_k + l_i q'' \tag{10}$$

where l_i is the total perimeter of the subelements which is crossed by q''. For the internal knots of the coolant the parameters g_{Hik} are identical to g_{Mik}, for those of the solid they must be calculated taking the ratio γ between the thermal conductivity in the solid and in the fluid into account. In steady state the conductive heat flux which enters the subelements in the transversal direction must be balanced by the convective heat flux in the longitudinal direction:

$$q'_{c_i} = \rho w_i c_p \frac{\partial T_c}{\partial z} \tag{11}$$

where w_i is the volume flow rate through the subelements around the knot i. The volume flow rate is zero for all the internal knots of the finned plate. For the other knots it can be calculated by integrating eqn (4) on the surface of the subelements around each knot. It then follows that

$$w_i = \sum_k g_{Aik} u_k. \tag{12}$$

On the cross-section the partial derivative of the temperature in the z-direction is constant and can be written as a function of q'':

$$\frac{\partial T_c}{\partial z} = \frac{q''(R+s)\beta}{\rho w_t c_p} \tag{13}$$

where w_t is the total volume flow rate through the conduit section. The convective heat flux then results:

$$q_{c_i} = \frac{q''(R+s)\beta}{w_t} \sum_k g_{Aik} u_k. \tag{14}$$

By letting q'_{k_i} be equal to q'_{c_i} for all the knots we obtain the following system:

$$\mathbf{G}_H * \mathbf{T} = \mathbf{D} \tag{15}$$

$$\mathbf{D} = \frac{q''}{k_c}\left(\frac{(R+s)\beta}{w_t}\mathbf{G}_A * \mathbf{U} - \mathbf{L}\right) \tag{16}$$

having gathered in matrices $\mathbf{G}_H$ and $\mathbf{G}_A$ the parameters g_{Hik} and g_{Aik}, respectively, in vector $\mathbf{T}$ the temperatures t_k of all the knots and in vector $\mathbf{L}$ the perimeters l_k. Vector $\mathbf{U}$ has now been extended to include the finned tube internal knot velocities, which are zero. Vector $\mathbf{T}$ can be partitioned by putting the temperature of an arbitrary knot (for example that with coordinates $(R+s, \beta)$, in which the maximum of the temperature is expected) in the subvector $\mathbf{T}_1$ and the other temperature in the subvector $\mathbf{T}_2$. Matrix $\mathbf{G}_T$ and vector $\mathbf{L}$ must be consequently partitioned. From eqn (16) we then obtain

$$\mathbf{G}_{H22} * \mathbf{T}_2 = \mathbf{D}_2 - \mathbf{G}_{H21} * \mathbf{T}_1. \tag{17}$$

By solving eqn (17) the temperature distribution in the portion of the cross-section is determined as a function of $\mathbf{T}_1$.

The bulk temperature can be calculated by assuming that the product of velocity and temperature in each element is approximated by an interpolation of the value which it assumes in the knots, with the same form factors of eqn (4). It then results in

$$T_b = \frac{1}{w_t} \sum_i \sum_k g_{Aik} u_k t_k \tag{18}$$

where the index i is extended to all the knots of the coolant.

Since the system is linear, an arbitrary value can finally be assigned to $\mathbf{T}_1$ in order to calculate the global heat transfer coefficient of the finned tube.

FINNED TUBE DESIGNING

Finned tubes are often used for removing heat fluxes which are imposed on the external surface. In such a situation the heat transfer optimization problem comprises dissipating as high a heat flux as possible, keeping the temperature of the external surface under a required value. The performances of the dissipative system must then be evaluated paying particular attention to the maximum drop between the temperature of the external surface and the bulk temperature of the coolant which flows inside the tube.

An appropriate global heat transfer coefficient for the system described in the previous section can be defined as follows:

$$h = \frac{q''}{T_{max} - T_b} \tag{19}$$

where T_{max} is the maximum temperature on the external surface. Moreover, it is possible to define an equivalent Nusselt number:

$$Nu_e = \frac{h2(R+s)}{k_c} \tag{20}$$

which corresponds to the Nusselt number which would be calculated if the same heat flux q'' were dissipated through a finless tube with a null wall thickness and radius equal to $R+s$. Such an equivalent Nusselt number is, for the finned tube, a function of the ratio γ.

It is interesting to observe that the heat flux, which can be removed by the finned tube per unit of length for a given drop between the external surface and bulk temperatures, depends only on the equivalent Nusselt number and does not change with the tube size. As a consequence, the heat flux dissipated per unit of surface is inversely proportional to the tube radius. A criterion for optimizing the fin relative dimensions and shape can then consist in maximizing Nu_e. In this way, for a given bulk temperature at a given coordinate z the finned tube dissipates the maximum heat flux per unit of length without exceeding the maximum temperature limit on the external surface at the coordinate z.

In many heat transfer optimization problems the dissipator is required to have as small a weight as possible. In finned tube designing it may then be useful to choose an unfinned tube with an established wall thickness as a reference and try to find the finned tube geometry which allows the highest equivalent Nusselt number to be obtained with the same weight of the reference tube for the same material, and hence with the same solid volume.

Moreover, in order to respect the temperature limit for an extended tube length it is expedient to let the bulk temperature slightly increase with the z coordinate. This is obtained by increasing the mass flow rate as much as possible. If the pressure drop between the beginning and the end of the tube is constrained, the maximum mass flow rate is obtained in correspondence with the lowest hydraulic resistance. After having optimized the tube geometry in order to maximize the heat flux which can be removed per unit of tube length, if the tube size is not constrained it is possible to obtain an established value for the hydraulic resistance by varying the tube radius. On the other hand, when the heat flux per unit of surface is required to be maximized, the tube size must be considered during the geometry optimization.

In order to evaluate the performances of finned tubes in terms of heat flux dissipated per unit of surface for the same hydraulic resistance and the same drop between the external surface and bulk temperature, a compared effectiveness can then be defined as the ratio between the global heat transfer coefficient of the finned tube and that of an unfinned tube with null wall thickness and the same hydraulic resistance of the first [13]:

$$E_c = \frac{h2R/M}{4.364k_c} \tag{21}$$

where M is a scale factor which can be calculated as the fourth root of the ratio between the hydraulic resistance of the finned tube and that of an unfinned tube with the same internal radius:

$$M = \sqrt[4]{\frac{(-\mathrm{d}p/\mathrm{d}z)}{2\pi w_t/\beta} \Big/ \frac{8\mu}{\pi R^4}}. \tag{22}$$

GEOMETRY OPTIMIZATION

Parameters R, s, β, a and the profile function $f(r)$ describe the geometry of the finned tube. Referring a, s and the radial coordinate to the inner radius it is possible to introduce the following variables, which do not depend on the tube size:

$$\alpha = \frac{a}{R}, \quad \sigma = \frac{s}{R}, \quad \eta = \frac{r}{R}, \quad \phi(\eta) = f(\eta R). \tag{23}$$

Let us assign a polynomial form to the profile function ϕ:

$$\phi(\eta) = \sum_{i=0}^{n} \psi_i \eta^i. \tag{24}$$

As fin profile-describing parameter the values of ϕ in $n+1$ equidistant points on the η-axis can be chosen:

$$\phi_i = \phi\left(1 - \frac{i}{n}\alpha\right) \quad \forall i = 0, 1, \ldots, n. \tag{25}$$

The polynomial function in fact is univocally determined by the values which it assumes in correspondence with $n+1$ values of η. Moreover, changes in ϕ_i induces in $\phi(\eta)$ variations of a more comparable entity than do changes in ψ_i. For this reason the first ones are preferable as fin profile-describing parameters instead of the latter. The area of the fin cross-section ξ and the average thickness of the finned tube wall $\bar{\sigma}$ then result:

$$\xi = 2\sum_{i=0}^{n} \psi_i(\phi_0, \ldots, \phi_n)\frac{1-(1-\alpha)^{i+2}}{i+2} \tag{26}$$

$$\bar{\sigma} = \sqrt{(1+\sigma)^2 + \frac{\xi}{\beta}} - 1. \tag{27}$$

Parameter $\bar{\sigma}$ is representative of the solid volume of the finned tube.

The geometry optimization problem consists now in finding the combination of parameters α, β, σ and ϕ_i which allow the maximum Nu_e or E_c to be obtained with respect to some constraining conditions. When only one parameter can be varied, as a consequence of the constraints, the optimum geometry can be determined with simple methods, which find, for example, the best parameter values by appropriate attempts. In the other cases the problem becomes more difficult. To solve it the following genetic algorithm [12, 14, 15] can then be successfully used.

The algorithm starts with a population of parameter combinations which have been generated by copying, with random mutations, the parameters of a prototype combination. For each combination of the initial population Nu_e or E_c are then calculated as an evaluation. Afterwards, an established percentage of the combinations is selected on the basis of the best evaluation. The parameters of the selected combinations are then reproduced with random mutation in order to generate a new population of the same proportions of the initial one. The combinations of the new population are valued and selected in the same manner as those of the parental population. The process is iterated until the evaluations no longer improve.

After reproduction some parameter can result as exceeding established limits related to the structural integrity of the system or to some particular requirement. Such a parameter must then be resized to the nearest acceptable value. Parameters ϕ_i must be resized together by considering the minimum $\phi_{\min}$ and the maximum $\phi_{\max}$ values which $\phi(\eta)$ assumes for η between $1-\alpha$ and 1:

$$\hat{\phi}_i = \begin{cases} \phi_{\max} - \dfrac{\phi_{\max}-\theta_{\min}}{\phi_{\max}-\phi_{\min}}(\phi_{\max}-\phi_i) & \text{if } \phi_{\min} < \theta_{\min} \\ \phi_{\min} + \dfrac{\theta_{\max}-\phi_{\min}}{\phi_{\max}-\phi_{\min}}(\phi_i-\phi_{\min}) & \text{if } \phi_{\max} > \theta_{\max} \end{cases} \quad \forall i = 0, 1, \ldots, n \quad (28)$$

$\hat{\phi}_i$ being the new parameter values, $\theta_{\min}$ and $\theta_{\max}$ the minimum and the maximum angles which are acceptable on the lateral fin profile.

When the optimization problem consists of finding the parameter combination which presents the highest Nu_e keeping the finned tube average wall thickness at an established value $\bar{\sigma}_0$ for a given number of fins, parameters α and ϕ_i can be reproduced with random mutations, while σ must be calculated as follows:

$$\sigma = \sqrt{(1+\bar{\sigma}_0)^2 - \frac{\zeta}{\beta}} - 1. \quad (29)$$

If σ results as being negative or too small, parameters ϕ_i can be opportunely resized or the parameter combination can be rejected by assigning it as a null evaluation.

When the problem consists of maximizing the heat flux per unit of length or surface for given solid volume and hydraulic resistance, an unfinned tube can be chosen as a reference. The reference radius R_r and wall thickness s_r must be established in order to respect the requirements. For a given number of fins, parameters α and ϕ_i can then be reproduced with random mutations. After having determined the velocity distribution in the finned tube cross-section referring to R_r, the scale factor M can be calculated. In order to present the required hydraulic resistance, the finned tube will have an internal radius equal to M times R_r. Taking this dimensioning into consideration, parameter σ can finally be determined as in the previous paragraph after having calculated the required value of $\bar{\sigma}_0$ on the basis of s_r and M.

RESULTS

Some optimizations of the finned tube geometry have been carried out with the genetic algorithm described in the previous section. In every case, populations of 20 samples and a selection percentage equal to 20 were established. During parameter reproduction uniformly distributed between -10 and $+10\%$ random errors were introduced. The genetic algorithm was stopped after 40 generations from that time in which an improvement was no longer observed.

In order to maximize the equivalent Nusselt number, for a number of fins equal to four, eight and 16 some preliminary trials have been carried out by choosing n equal to 0 and leaving all the other parameters free to change. As a prototype a finned tube with α equal to 0.5, σ equal to 0.1 and ϕ_0 equal to $\beta/2$ was employed. As it was intuitively expected the algorithm tried to extend the normalized height of the fin α as much as possible in order to create separated narrow channels. Aiming to allow a more uniform distribution of the coolant in the tube, in the subsequent optimizations α has been constrained to 0.8 and $\phi(\eta)$ has been imposed to be no greater than 0.95β. Moreover, to ensure the structural integrity of the finned tube, σ has to be imposed to be no less than 0.05 and $\phi(\eta)$ no less than 0.05β. In this way the minimum acceptable width of the fin at the base results as being greater than at the tip.

In Fig. 2 some finned tube geometries which maximize Nu_e are shown for n equal to 0, 2 and 4, β equal to $\pi/4$ and γ equal to 500. The high value of β allows changes in the lateral fin profile to be better appreciated. The value chosen for γ corresponds to the case of a finned tube made of copper and cooled by water. The describing parameters of the optimum geometries are reported in Table 1 together with the average finned tube thickness, the equivalent Nusselt number, the compared effectiveness and factor M, which is

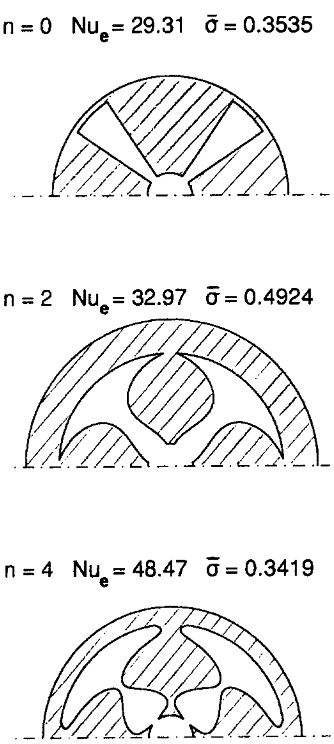

Fig. 2. Finned tube geometries which maximize Nu_e when α is equal to 0.8, β to $\pi/4$ and γ to 500.

Table 1. Characteristic parameters of some optimum finned tube geometries. Cases are listed in the same order they are discussed in the text

β	γ	n	$\bar{\sigma}$	σ	ϕ_0	ϕ_1	ϕ_2	ϕ_3	ϕ_4	Nu_e	E_c	M
$\pi/4$	500	0	0.354	0.05	0.5969	—	—	—	—	29.31	1.97	3.26
$\pi/4$	500	2	0.492	0.299	0.0621	0.6234	0.175	—	—	32.97	2.18	2.66
$\pi/4$	500	4	0.342	0.151	0.1515	0.4325	0.7157	0.231	0.5848	48.47	3.54	2.73
$\pi/4$	500	0	0.3	0.05	0.4806	—	—	—	—	23.84	2.1	2.48
$\pi/4$	500	2	0.3	0.075	0.071	0.6135	0.1645	—	—	32.26	2.62	2.63
$\pi/4$	500	4	0.3	0.111	0.171	0.3915	0.7307	0.2251	0.5517	48.06	3.71	2.67
$\pi/4$	500	0	0.2	0.05	0.2761	—	—	—	—	19.74	2.33	1.85
$\pi/4$	500	2	0.2	0.05	0.0401	0.3928	0.0502	—	—	23.13	2.57	1.96
$\pi/4$	500	4	0.2	0.051	0.2937	0.2203	0.561	0.207	0.3076	34.49	3.46	2.17
$\pi/4$	500	0	0.1	0.05	0.0879	—	—	—	—	18.16	2.54	1.56
$\pi/4$	500	2	0.1	0.05	0.0408	0.0922	0.2706	—	—	18.54	2.53	1.6
$\pi/4$	500	4	0.1	0.05	0.0584	0.0401	0.1005	0.1945	0.0653	19.02	2.61	1.59
$\pi/4$	50	0	0.3	0.05	0.4806	—	—	—	—	20.9	1.84	2.48
$\pi/4$	50	2	0.3	0.05	0.3144	0.5665	0.2778	—	—	23.66	1.95	2.64
$\pi/4$	50	4	0.3	0.064	0.3302	0.4621	0.7053	0.3408	0.7377	30.51	2.29	2.87
$\pi/8$	500	0	0.3	0.078	0.216	—	—	—	—	53.87	3.94	2.91
$\pi/8$	500	4	0.3	0.126	0.069	0.2315	0.2506	0.047	0.3461	74.03	5.32	2.83
$\pi/16$	500	0	0.3	0.212	0.0452	—	—	—	—	94.57	6.55	2.73
$\pi/16$	500	4	0.3	0.198	0.0426	0.0702	0.0479	0.0104	0.1678	107.7	7.19	2.87
$\pi/4$	500	0	0.203	0.05	0.2827	—	—	—	—	19.81	2.32	1.87
$\pi/4$	500	2	0.193	0.05	0.0408	0.3721	0.0453	—	—	22.5	2.56	1.92
$\pi/4$	500	4	0.182	0.051	0.0981	0.1949	0.4552	0.3089	0.0441	26.96	2.97	1.98

representative of the hydraulic resistance. The table also contains the characteristic parameters of the optimum geometries which will be subsequently discussed.

It is evident that under the above-considered conditions the adoption of a lateral fin profile, along which the angular coordinate varies as a polynomial function of the radial coordinate, provides noticeable advantages in terms of heat transfer and required solid volume. The equivalent Nusselt number of the tube with the optimum fourth polynomial order lateral fin profile is in fact more than 1.6 times that of the tube with the optimum constant profile angle. Moreover, the first geometry requires a solid volume which is a little less than that of the latter. Even with a second-order fin profile noticeable improvements are obtainable in the heat transfer, but a greater solid volume is required.

It is interesting to observe how the optimum finned tube performances change by limiting the available solid volume. In Fig. 3 the geometries which maximize Nu_e when n is equal 0, 2 and 4 and $\bar{\sigma}$ is equal to 0.3, 0.2 and 0.1 are shown. By constraining the average wall thickness to 0.3 the relative improvements of the higher order lateral fin profile increase. The equivalent Nusselt number of the zero-order profile noticeably decreases, while that of the second-order profile is affected by a small reduction, although its average wall thickness has been strongly lowered. The reduction of the average wall thickness to 0.2 causes significant decrements in Nu_e for every polynomial order, but the higher order profiles still perform much better. When $\bar{\sigma}$ is reduced to 0.1 the equivalent Nusselt number does not decrease much for the zero-order profile, while for the other order profiles it falls to low values which are near that of the first one.

Such a relationship between the available solid volume and the maximum equivalent Nusselt number which can be obtained with the different polynomial orders can be explained by considering that the heat transfer improvements depend on the extension of the fin surface, but mainly on the alteration of the flow induced by the fin shape. In particular, the optimum finned tube geometries are the result of a compromise between two main thermofluidodynamical exigences. The first consists of having, in the spaces between the fins, velocities which are comparable with those at the fin tip or higher. High velocities near the fin tip would cause in fact high thermal gradients in this region, which would lower the bulk temperature without enhancing the heat transfer from the unfinned part of the tube wall and from the lateral surface of the fin, which is much more extended than the tip transversal one. The second exigence consists of maintaining the velocity maximum as near the solid surface as possible, in order to relatively increase the thermal gradient. As a consequence of the first exigence the channels which are created between the fins must not be too narrow, and for the second one not too large. The higher polynomial order profiles, which are more articulate, can better satisfy both exigences, but when the available solid volume is too small to create sufficiently narrow channels they can only conform to the first one.

This phenomenon can be better understood by observing the velocity distribution in the optimum finned tube with second- and third-order lateral profile

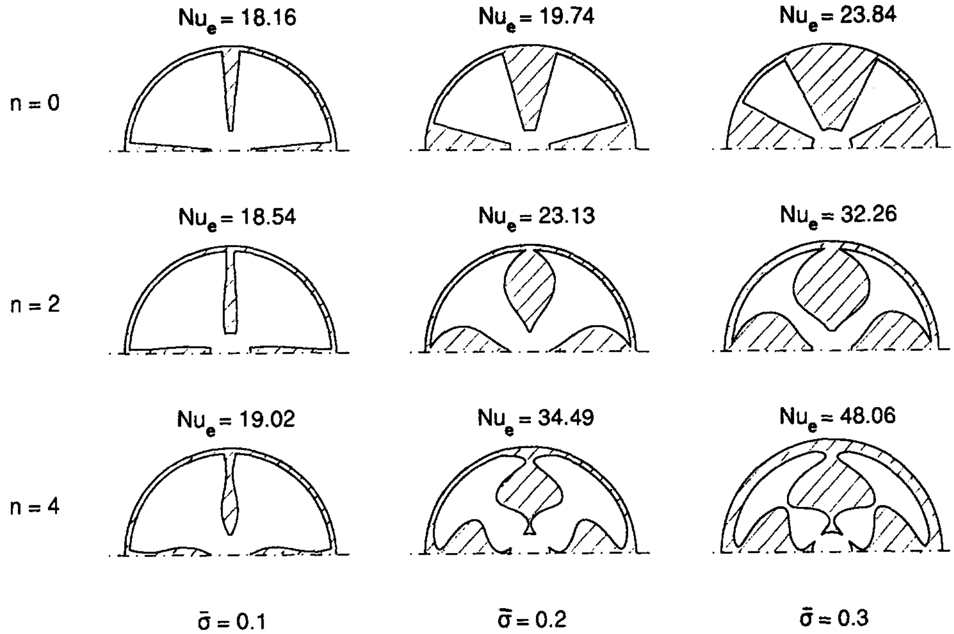

Fig. 3. Finned tube geometries which maximize Nu_e when α is equal to 0.8, β to $\pi/4$, γ to 500 and $\bar{\sigma}$ is constrained to different values.

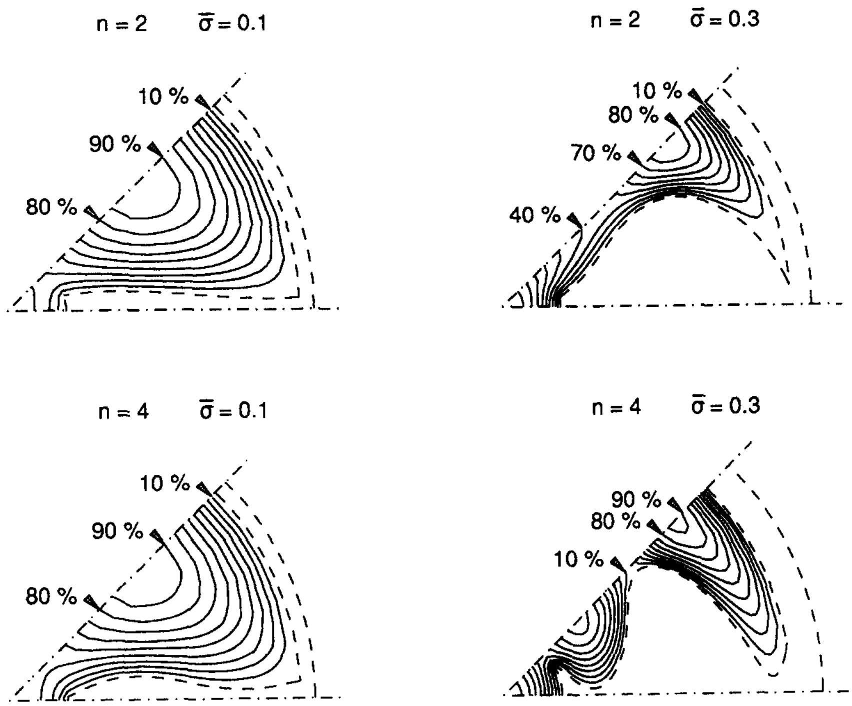

Fig. 4. Velocity distributions in the cross-section in some of the cases of Fig. 3. Curves are drawn every 10% of the maximum velocity.

fins. In the portion of the cross-section shown in Fig. 4 corresponding to the case of second-order profile fins with sufficiently high amounts of solid volume two channels can be distinguished, at the center and close to the wall of the tube, in which the maximum velocities reach comparable values. Under the same conditions the third-order profile, which is more articulate, creates three channels with comparable maximum velocities, in which the temperature gradients are relatively higher (Fig. 5). When the available

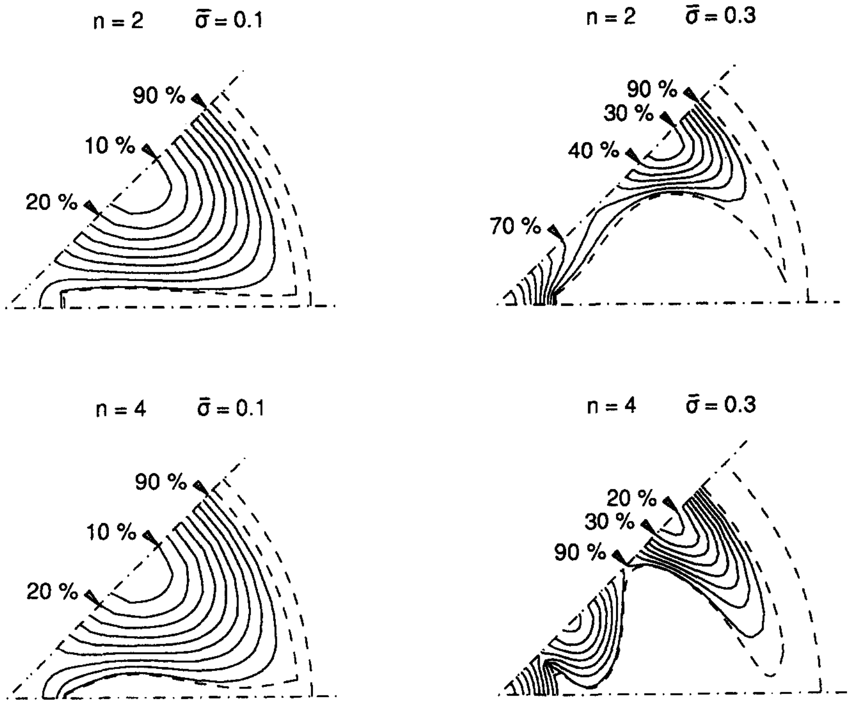

Fig. 5. Temperature distributions in the cross-section in the same cases of Fig. 4. Curves are drawn every 10% of the difference between the maximum and minimum temperatures.

solid volume is too small, since sufficiently narrow channels cannot be created, the exigence of obtaining higher velocities in the space between the fins than at the tube center prevails (Fig. 4).

By varying the available solid volume it could be observed that in general there is no monotonic relationship between this parameter and the heat flux which can be dissipated by a fin profile of an established order. For this reason, contrary to what occurs under different conditions, it is not possible to define a comparable effectiveness of the fin as the ratio between the heat flux removed and that dissipable with another fin with a reference shape and with the same volume of the first.

It is important to observe that the fourth-order profile allows the lateral fin surfaces to be better exploited in terms of heat transfer than does the second-order profile. When $\bar{\sigma}$ is equal to 0.3 the low density of isothermal curves in the space between the beginning of the fin and the tube wall (Fig. 5) demonstrates that a very low heat flux is extracted from this region. In particular, the coolant nearly stands still (Fig. 4) and the heat passes through it from the tube wall to the fin. The local convective heat transfer coefficient is then negative at the beginning of the fin. This phenomenon is avoided by the fourth polynomial order optimum finned tube. Therefore, the local heat transfer coefficient, under similar conditions, is very sensitive to the fin shape and a mathematical model which assumes this parameter as a constant cannot be successfully employed for fin profile optimization.

Figure 6(a) compares the Nusselt numbers of the optimum geometries obtained for n equal to 0, 2 and 4, σ constrained to 0.3 and γ equal to 50 and 500. It can be observed that when γ is lower for a magnitude order the improvements in the heat transfer are smaller, but the higher order profile fins still perform significantly better than the zero-order ones.

The Nusselt numbers of the optimum geometries obtained for γ equal to 500, σ constrained to 0.3 and β equal to $\pi/4$, $\pi/8$ and $\pi/16$ are compared together in Fig. 6(b). For β equal to $\pi/8$ the fourth-order fin maintains a shape which is similar to that assumed by the fin for β equal to $\pi/4$. When β is equal to $\pi/16$, since the coolant velocity at the tube center is much higher than between the fins the exigence of enlarging the channel in this region prevails, and the fin results in being flatter (Table 1). If the fin height were greater and the channel at the tube center narrower, the optimum fourth-order fin could be more ondulate and efficient.

The improvements in the equivalent Nusselt number of the above-discussed finned tube geometries do not always result in an increase of the hydraulic resistance. It is then interesting to optimize the geometry whilst trying to maintain the hydraulic resistance at low values, as in the following examples.

The characteristic parameters of the finned tube geometries which maximize the dissipated heat flux per unit of length with the same hydraulic resistance and the solid volume of a reference unfinned tube whose wall thickness is 0.6 times its radius are reported

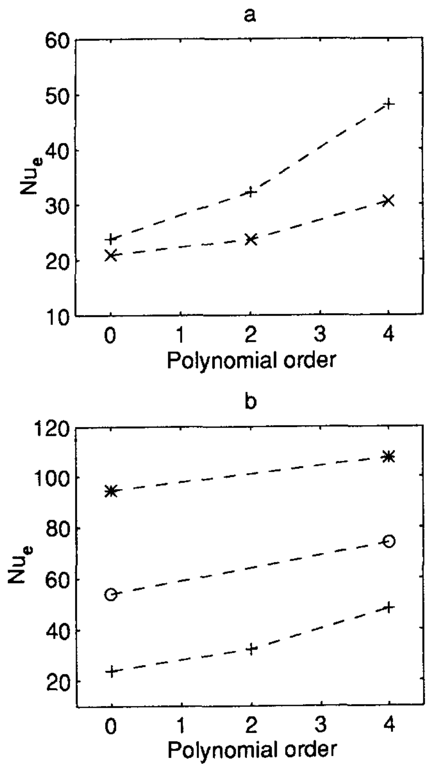

Fig. 6. Comparison between the Nusselt numbers of the optimum finned tube geometries obtained by letting γ be equal to 500 (+) and 50 (×) and keeping β equal to $\pi/4$ (a) and by letting β be equal to $\pi/4$ (+), $\pi/8$ (○) and $\pi/16$ (*) and keeping γ equal to 500 (b) with α equal to 0.8 and $\bar{\sigma}$ constrained to 0.3.

in the last three rows of Table 1. The higher order profiles still perform noticeably better and the increase in the scale factor M are limited. The same optimum geometries also result by maximizing, under the same conditions, the dissipated heat flux per unit of surface.

CONCLUSIONS

The results obtained demonstrate that it is possible to noticeably increase the heat transfer effectiveness of an internally finned tube by assigning to the fin an ondulate lateral profile such as that described by a polynomial function, when the available solid volume is sufficiently high. In the case of a finned tube made of copper and cooled by water with an average wall thickness equal to 0.3 times the internal radius the optimum fourth polynomial order profile provides an increase of more than 100% in the dissipated heat flux per unit of length with respect to the optimum tube zero-order profile. Minor reductions of the available solid volume do not affect the improvements of the polynomial fin profiles significantly, but when this parameter is reduced to very low values such improvements become almost null.

The increments in the dissipated heat flux due to the adoption of an ondulated lateral fin profile are higher when the ratio between the thermal conductivity of the solid and the fluid is high and when the fins are more widely spaced. In the studied cases the higher order fin profiles do not always cause a greater hydraulic resistance than the zero-order one. When that occurs, the disadvantage related to the increase in the hydraulic resistance is largely overcome by improvements in the heat transfer. Therefore, the geometries which provide the greatest Nusselt numbers also present the highest compared effectiveness.

The proposed optimization algorithm appears to be a useful tool to solve the problem of finding the optimum finned tube geometry with regards to the dissipated heat flux per unit of length or surface. It can be used to fit the parameters of more complex fin profile-describing functions. A polynomial form has been considered in the present work, because it allows sufficiently articulate profiles to be obtained without complicating the finned tube production too much. Since the greatest improvements in the heat transfer of the found optimum geometries appear to be derived from the fluidodynamical more than from the conductive fin characteristics, it would be interesting to test the performances of fins containing empty spaces.

Internally finned tubes with the optimum geometries which have been presented in this work can be produced by extrusion of melted metal through an appropriate die and subsequent rectification by means of a cursor which expands the tubes and gives the fins their final shape.

By considering temperature-dependent parameters, a more rigorous solution could be obtained for the problem of optimizing the heat transfer in internally finned tubes under laminar coolant flow conditions. For the case in which longitudinal conduction slightly influences the heat transfer, the proposed mathematical model could still be utilized by solving eqns (9) and (17) iteratively. The same optimization algorithm could probably be successfully employed.

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